Perturbation spectrum in inflation with cutoff

Achim Kempf
Institute for Fundamental Theory
Departments of Physics and Mathematics
University of Florida, Gainesville, FL 32611, USA
Email: kempf@phys.ufl.edu

Jens C. Niemeyer
Max-Planck-Institut für Astrophysik
Karl-Schwarzschild-Str. 1, D-85748 Garching, Germany
Email: jcn@mpa-garching.mpg.de

Abstract

As a potentially accessible window to aspects of Planck scale physics it has been pointed out that the perturbation spectrum predicted by inflation may be sensitive to a natural ultraviolet cutoff. A fairly general classification of the possible short-distance cutoffs that one may encounter at the Planck scale has also recently been given. Indeed, various studies of quantum gravity and string theory point towards one of the types of cutoff in this classification. This cutoff has been implemented into the standard inflationary scenario. We here continue this approach by investigating its effects on the predicted perturbation spectrum. We find that the size of the effect depends crucially on the scale separation between cutoff and horizon scales during inflation, becoming negligibly small if the cutoff scale is as small as the Planck length.

1 Introduction

During the inflationary phase of the very early universe (see e.g. [1] for an overview) space-time is assumed to expand in a quasi-exponential fashion. In the comoving frame, quantum fluctuations of the inflaton field are continuously redshifted until their wavelength equals the physical horizon distance, whereupon they become "frozen" until they re-enter the Hubble volume during the ensuing radiation or matter dominated epochs. These fluctuations are thought to be responsible for seeding the temperature fluctuations of the cosmic microwave background radiation (CMBR) and the gravitational clustering of matter, whose statistical properties may therefore provide a window into the realm of high-energy physics.

Crucially, in the case of a sufficiently long period of inflation, all of the scales of cosmological interest today correspond to wavelengths below the Planck length early on when the initial conditions are prescribed. Therefore, inspired by similar studies in the context of Hawking radiation, a series of papers [2, 3, 4, 5] has investigated the sensitivity of the predictions of inflationary scenarios with respect to changes of trans-Planckian physics. Those studies encoded transplanckian physics in a simple way as nonlinearities of the dispersion relation of the Fourier mode functions.

Since linearity of the field and hence Gaussianity of the fluctuations remains unchanged, the potential consequences of such modifications are limited to a possible scale dependence of the power spectrum and a possible change in its overall amplitude. It was shown [5] that under rather general conditions on the dispersion relation no observable effects can be expected, although Ref. [2] reaches a somewhat different conclusion. However, those studies suffer from fundamental limitations. First of all, with the exception of Ref. [4] all of the employed dispersion relations were chosen *ad hoc* so as to provide bounds on the frequency, wavelength or both without reference to an underlying theory. More importantly, the question of mode generation, i.e. how each semiclassical quantum field degree of freedom emerges out of the Planck regime, has not been addressed.

In contrast, Ref. [6] proposes a scenario where the UV cutoff is provided by a modified quantum mechanical commutation relation that effectively limits the experimentally attainable resolution of small spatial distances. This UV cutoff is one of very few types of short-distance structures that appear in the classification of short distance structures presented in [7] which applies to all quantum gravity theories that effectively represent each dimension by a linear operator. Indeed, corresponding short-distance uncertainty relations of this kind have appeared in various studies of quantum gravity and string theory, see e.g.[8]. In Ref. [6] this short distance cutoff has been implemented into the theory of a minimally coupled scalar field living in an expanding Friedmann-Robertson-Walker (FRW) background and it has been shown how the decoupling degrees of freedom are continuously generated dynamically at the time of their "Planck scale crossing". Here, we aim to extend the analysis of [6] by estimating the magnitude of any corrections to the standard predictions for the statistical distribution of inflationary perturbations arising from the modified short-distance behavior.

The approach in Ref. [6] which we will follow here utilizes that, as has been shown in [9], the quantum gravity and stringy uncertainty relation cutoff (see e.g. [8]) can be effectively modeled by corrections to the commutation relations:

$$[\mathbf{x}, \mathbf{p}] = i(1 + \beta \mathbf{p}^2) \tag{1}$$

and its higher dimensional generalizations, which are unique to first order in beta, see [10]. Indeed, it is easy to check that such a correction gives rise to a lower bound $\Delta x_{min} = \sqrt{\beta}$ for distance measurements. A Hilbert space representations of Eq. (1) is given by introducing the auxiliary variable ρ which is essentially the momentum variable p but differs from it at small distances, i.e. at distances close to Δx_{min}. While this is initially a quantum mechanical structure, it can be implemented into quantum field theory, see Ref. [6]. Within the scalar quantum field theory on an inflationary background as defined in Ref. [6] one finds that, interestingly, those variables, $\tilde{k}$, in which the mode equations decouple, no longer strictly coincide with the comoving momentum variables, k, although they do of course approximately coincide for small k, i.e. for large distances. Conversely, this means that the comoving momentum modes now decouple only when they have grown to large proper distances and that the comoving momentum modes do couple initially when they emerge from the cutoff scale. For the quantum theory of the actual mode creation mechanism see again Ref. [6]. Explicitly, one obtains within this framework the following mode equation for the decoupling $\tilde{k}$-modes:

$$\phi''_{\tilde{k}} + \frac{\nu'}{\nu}\phi'_{\tilde{k}} + \left(\mu - 3\left(\frac{a'}{a}\right)' - 9\left(\frac{a'}{a}\right)^2 - \frac{3a'\nu'}{a\nu}\right)\phi_{\tilde{k}} = 0 \ . \tag{2}$$

Here, a is the scale factor of the FRW line element and we defined the functions

$$\mu(\eta, \tilde{k}) \ := \ -\frac{a^2 \mathrm{plog}(-\beta\tilde{k}^2/a^2)}{\beta\left(1 + \mathrm{plog}\left(-\beta\tilde{k}^2/a^2\right)\right)^2} \tag{3}$$

$$\nu(\eta, \tilde{k}) \ := \ \frac{e^{-\frac{3}{2}\mathrm{plog}\left(-\beta\tilde{k}^2/a^2\right)}}{a^4\left(1 + \mathrm{plog}\left(-\beta\tilde{k}^2/a^2\right)\right)} \tag{4}$$

that utilize the “product log” plog, which is the inverse of the function $x \to xe^x$. The solutions are automatically defined only from a finite value of η, i.e. every mode possesses its “creation time”. It is the time when, in terms of proper distances, the mode equals the size of the cutoff scale, i.e. it is the conformal time η_c defined implicitly by

$$a(\eta_c) = \tilde{k}\sqrt{e\beta} \sim \tilde{k}\,\Delta x_{min} \ . \tag{5}$$

At the creation time, the differential equation possesses what is called an irregular singular point. To see this, note that the function plog which enters the differential equation through the functions μ and ν is not analytic at the creation time. Below, we will further discuss possible implications for the choice of initial conditions and therefore for the choice – or possible uniqueness – of the initial vacuum.

All physical observables in our model universe can be expressed in terms of $\tilde{k}$ instead of the usual Fourier variable k. This argument applies also to the transfer function $T(t, \tilde{k})$ which relates today's observable perturbations to the horizon crossing amplitude of $\phi_{\tilde{k}}$, provided that the perturbation amplitudes are measured as a function of $\tilde{k}$. In practice, these measurements are carried out on cosmological scales where $\tilde{k} = k$ to extremely good accuracy, so we do not expect any consequences from the re-labelling of physical observables such as the angular size distribution of CMBR fluctuations. In other words, any statement about the scale dependence and Gaussianity of the horizon crossing amplitudes of $\phi_{\tilde{k}}$ translate into corresponding statements about CMBR fluctuations, at least to the same extent as in the standard theory. Let us note, however, that this would not be true if the cutoff had different properties for different fields, e.g. if linear metric fluctuations behave differently on small scales than the inflaton field. The following analysis assumes that this is not the case[1].

Eq. (2) is linear in $\phi_{\tilde{k}}$ so that Gaussianity of the distribution of fluctuations in $\tilde{k}$-space is protected. Consequently, we expect no deviations from Gaussianity owing to the proposed modifications of the short-distance behavior. We can therefore restrict attention to examining possible new effects on the scale dependence and overall amplitude of the power spectrum.

2 Analysis in oscillator variables

It turns out to be very convenient to change from the field variables used in Ref. [6] to slightly new variables defined by:

$$\varphi_{\tilde{k}} \equiv \nu^{1/2} \phi_{\tilde{k}} \ . \tag{6}$$

Indeed, while the mode equation Eq. (2) in terms of the original field ϕ is of the type of a harmonic oscillator with friction, there is no friction term in the mode equation when written in terms of the new variable φ:

$$\varphi''_{\tilde{k}} + \omega^2(\eta)\varphi_{\tilde{k}} = 0 \tag{7}$$

where $\omega(\eta)$ obeys the time dependent, nonlinear dispersion relation

$$\omega^2(\eta) = \mu - 6\left(\frac{a'}{a}\right)^2 + \left(\frac{\nu'}{2\nu}\right)^2 - \frac{3(a'\nu' + a''\nu)}{a\nu} - \frac{\nu''}{2\nu} \ . \tag{8}$$

[1] At horizon crossing of the mode $\tilde{k}$, i.e. when $\tilde{k} \approx aH$, we also note that $\tilde{k}^2$ and k^2 differ only by the constant factor $-\sigma^{-2}\text{plog}(-\sigma^2)$, independent of k, where σ is defined in Eq. (13).

The Wronskian condition from Ref. [6] now also simplifies to

$$\varphi_{\tilde{k}}\varphi_{\tilde{k}}^{*\prime} - \varphi_{\tilde{k}}'\varphi_{\tilde{k}}^{*} = i \tag{9}$$

as usual. Note also that if we denote the standard field mode with a vanishing minimum position uncertainty as $\chi_{\tilde{k}} = \varphi_{\tilde{k}}(\beta \to 0)$, we obtain the usual equation of motion for the $\tilde{k}$ mode of a free, minimally coupled scalar field in an expanding FRW space-time, where χ is in the conventions of e.g. [11] [2]:

$$\chi_{\tilde{k}}'' + \omega_0^2 \chi_{\tilde{k}} = 0 \tag{10}$$

with

$$\omega_0^2 = \tilde{k}^2 - \frac{a''(\eta)}{a(\eta)} \ . \tag{11}$$

Again, there is the question of initial conditions for $\varphi_{\tilde{k}}$ that determine the vacuum for $\hat{\phi}$. Ideally, regularity arguments at the irregular singular point of the mode equation, encountered at the creation time η_c for each mode, should fix the choice. We do not have a definite answer at this point, but asymptotic methods will shed some light on the situation. Some indications of vacuum fixation by regularity arguments are sketched in the Conclusions. Indeed, a solution of the singularity problem is not strictly necessary for the present analysis. It will be shown below that the evolution of $\varphi_{\tilde{k}}$ is essentially adiabatic from a certain time η_i onwards, i.e. for all $\eta \geq \eta_i \equiv \eta_c(1+\epsilon)$ where ϵ can but need not be a small number. The state of $\hat{\phi}$ at $\eta \geq \eta_i$ can be determined by consistency arguments to be the adiabatic vacuum (e.g.,[11])

$$\varphi_{\tilde{k}}(\eta) = \frac{1}{\sqrt{2\omega(\eta)}} \exp\left(-i \int_{\eta_i}^{\eta} \omega(\tilde{\eta}) d\tilde{\eta}\right) \ , \tag{12}$$

where the normalization follows from Eq. (9). This is because, as argued in Refs. [12, 5], any small deviation from the adiabatic vacuum close to the Planck scale would likely suppress inflation altogether due to back-reaction of the energy density contained in $\varphi_{\tilde{k}}$ on the cosmic expansion. In order to be consistent with the assumptions of Ref. [6] (i.e., negligible back-reaction), any admissible initial condition needs to converge onto the adiabatic vacuum as soon as the latter is well defined.

3 Adiabatic analysis

Eq. (7) belongs to the class of harmonic oscillator equations featuring a dispersion relation that is asymptotically linear for small physical wavenumbers but becomes nonlinear at high wavenumbers (small wavelengths). In the context of cosmology, such

[2] Of course, also $\tilde{k} = k$ for $\beta \to 0$. However, for the reasons explained above we prefer to label everything in terms of $\tilde{k}$ in order to avoid discussing the $\tilde{k} \to k$-map.

systems were investigated in Refs. [2, 3, 5], and in the framework of Hawking radiation many times before (see [13] for references). Unlike in the above references, where the dispersion relation was typically tailored *ad hoc* to fit some desired shape, Eq. (8) followed directly from a general study of realistic short-distance structures of space-time and may therefore perhaps be considered more fundamental (see also Ref. [4] for a similar approach).

It is useful to express the separation between the cutoff scale (here parameterized by $\beta^{1/2}$) and the inflationary horizon scale in terms of the dimensionless parameter σ defined as

$$\sigma \equiv \beta^{1/2} H \ . \tag{13}$$

If $\sqrt{\beta} \sim \Delta x_{min}$ is identified with the Planck length, the amplitude of temperature fluctuations of the cosmic microwave background indicates that $\sigma \sim 10^{-5}$ at the time when the presently observable scales left the horizon during inflation.

In order to generalize the notion of "horizon crossing" to our non-standard equation of motion, we Taylor-expand Eq. (8) around $\sigma = 0$ and find that $\omega(\eta)^2 = \omega_0^2 + O(\sigma^2)$. Correspondingly, the usual definition of horizon crossing in terms of $\tilde{k} = aH$ is valid to within the same accuracy.

We are interested in sources of deviation from the standard (i.e., $\beta \to 0$) result for the scale dependence and overall normalization of the horizon crossing amplitude of $\phi_{\tilde{k}}$. Following Sec. (2), we need to compare the amplitudes of $\varphi_{\tilde{k}}$ and $\chi_{\tilde{k}}$ at the horizon crossing time η_h, which is when $\tilde{k} \approx a(\eta_h)H$ [3].

One possible signature of the cutoff in the spectrum is due to non-adiabatic particle production during the evolution from η_i to η_h, which may give rise to a modulation of $\varphi_{\tilde{k}}(\eta_h)$ around the amplitude predicted for $\beta \to 0$ [5]. This may, in turn, be reflected by a breaking of scale invariance of the perturbation power spectrum. The *relative* magnitude of this effect, denoted in Ref. [5] as β_k, can be shown to be bounded by the maximum of the adiabaticity parameter

$$\mathcal{C}(\eta) = \left| \frac{\omega'(\eta)}{\omega^2(\eta)} \right| \ . \tag{14}$$

Fig. 1 displays the adiabaticity parameter $\mathcal{C}$ computed from Eq. (8) for the case $\sigma = 10^{-5}$ and $\tilde{k} = 1$ as a function of time, beginning at $\eta_i = \eta_c(1+\epsilon)$, where we arbitrarily chose $\epsilon = 10^{-5}$. Evidently, non-adiabaticity is negligible under these conditions. Defining the initial state closer to η_c corresponds to increasingly weaker bounds on β_k, until adiabaticity breaks down altogether at the singularity. However, as argued in Sec. (2), self-consistency demands the solution to converge onto the adiabatic vacuum as soon as it is well defined (i.e., as soon as $\mathcal{C} \ll 1$) and Fig. (1) shows this to be the case at $\eta_i = \eta_c(1 + 10^{-5})$.

Having shown that scale invariance is preserved if σ is small, we need to consider the overall amplitude of the power spectrum. Taking the adiabatic solution Eq. (12)

[3] Equivalently, we could compare $\phi_{\tilde{k}}$ and $a^2\chi_{\tilde{k}}$, as $\nu \to a^{-4}$ for $\beta \to 0$.

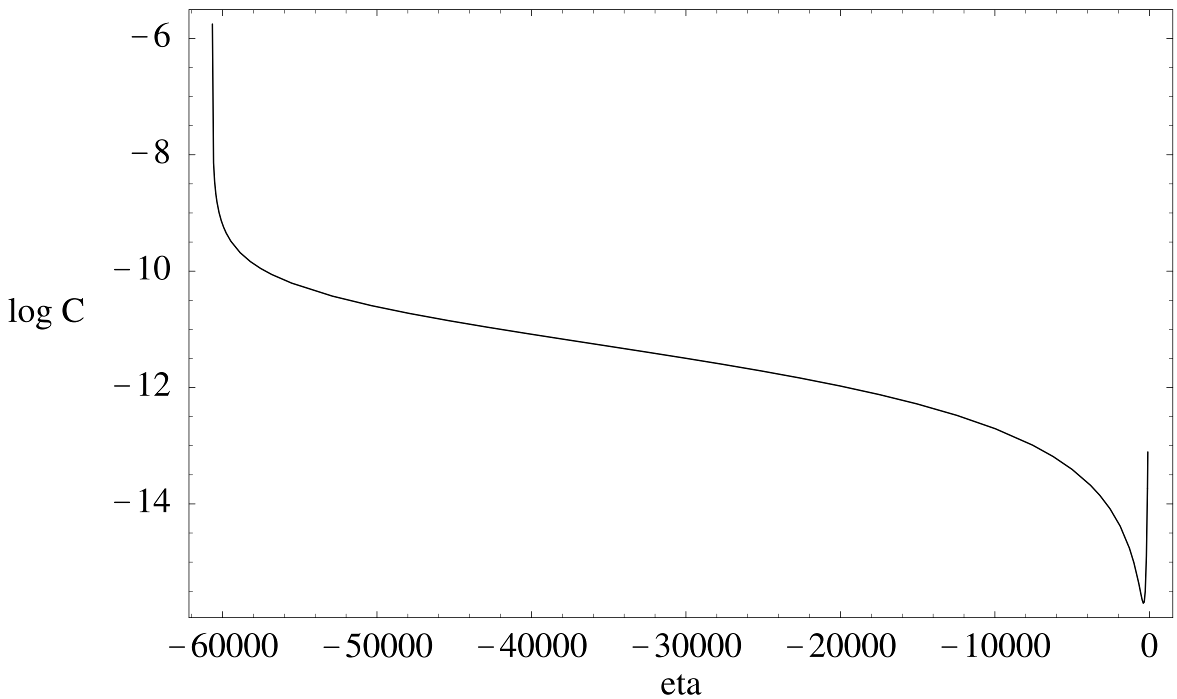

Figure 1: Logarithm of the adiabaticity parameter $\mathcal{C}$ for $\sigma = 10^{-5}$ and $\tilde{k} = 1$, beginning at $\eta_i = \eta_c(1+\epsilon)$, for $\epsilon = 10^{-5}$. The sudden rise of $\mathcal{C}$ for $\eta \to 0$ signals the usual onset of non-adiabaticity due to cosmic expansion.

as a reasonable approximation to the exact functions $\varphi_{\tilde{k}}(\eta)$ and $\chi_{\tilde{k}}(\eta)$ on length scales larger than the cutoff but smaller than the horizon scale (where expansion violates adiabaticity), i.e. for times $\eta_i \ll \eta \ll \eta_h$, one finds that

$$D(\eta) \equiv \frac{\varphi_{\tilde{k}}(\eta)}{\chi_{\tilde{k}}(\eta)} = \left(\frac{\tilde{k}}{\omega(\eta)}\right)^{1/2} . \tag{15}$$

A good estimate for the impact of the nonlinear dispersion relation on the amplitude of the power spectrum is obtained by noting that this expression for $D(\eta)$ remains approximately valid until η_h, since cosmic expansion affects both solutions in roughly the same way. It is readily verified in this case that $D(\eta_h) = 1 + O(\sigma^2)$. Hence, the impact of the cutoff on the perturbation spectrum depends crucially on the separation between the cutoff and the Hubble scale, being negligible if $\sigma \ll 1$.

4 Scaling analysis

The scaling behavior of the perturbation spectrum can also be investigated by studying the scaling behavior of the wave equation Eq. (2). Let us begin by considering the case of an exactly de Sitter type expansion. In this case, we expect that time translation invariance is broken neither by our introduction of a cutoff nor by the background expansion. We therefore expect a scale invariant perturbation spectrum.

Indeed, we first observe that if $\phi_{\tilde{k}}(\eta)$ is a solution to the $\tilde{k}$ mode equation and r is any arbitrary positive number then $\phi_{\tilde{k}}(r\eta)$ is a solution of the mode equation for the mode $r\tilde{k}$. This is straightforward to verify and it is of course also true for the usual inflationary scenario without a cutoff.

The solutions $\phi_{r\tilde{k}}(\eta)$ that are obtained in this way by scaling the solution $\phi_{\tilde{k}}(\eta)$ all obey of course the same initial conditions. We can also conclude that if η is a special time for the solution $\phi_{\tilde{k}}$, then, correspondingly, η/r is a special time for the solution $\phi_{r\tilde{k}}$. For example, if we denote the creation and the horizon crossing times of the mode $\phi_{\tilde{k}}$ by η_c and η_h, then the mode $\phi_{r\tilde{k}}(\eta)$ possesses the creation and the horizon crossing times $\eta_c(r\tilde{k}) = \eta_c/r$ and $\eta_h(r\tilde{k}) = \eta_h/r$.

Let us further assume that the solution $\phi_{\tilde{k}}(\eta)$ is normalized with respect to the Wronskian condition. We also need that all the solutions $\phi_{r\tilde{k}}(\eta)$ are normalized with respect to the Wronskian condition for the respective $r\tilde{k}$ modes. As is straightforward to verify, the ansatz

$$\phi_{r\tilde{k}}(\eta) = N(r)\phi_{\tilde{k}}(r\eta) \tag{16}$$

yields

$$N(r) = r^{3/2} \tag{17}$$

so that $\phi_{r\tilde{k}}(\eta) = r^{3/2}\phi_{\tilde{k}}(r\eta)$, and therefore:

$$\phi_{r\tilde{k}}(\eta/r) = r^{3/2}\phi_{\tilde{k}}(\eta) \tag{18}$$

Choosing for η the horizon crossing time of the $\tilde{k}$ mode we now obtain how the horizon crossing amplitude scales when scaling the decoupling momentum

$$\phi_{r\tilde{k}}(\eta_h(r\tilde{k})) = r^{3/2}\phi_{\tilde{k}}(\eta_h) \tag{19}$$

which means:

$$\phi_{r\tilde{k}}(\text{horizon crossing}) \sim r^{3/2} \tag{20}$$

In order to make contact with the conventions in the literature, let us now recall that, usually, field variables $\psi(\eta, k)$ in comoving momenta k are obtained by first scaling from proper position coordinates to comoving position coordinates and then, second, by Fourier transforming to the comoving momentum. In [6], however, we obtained fields $\phi(\eta, k)$ over comoving momenta k by first Fourier transforming from proper positions to proper momenta and then, second, by scaling to comoving momenta. However, scaling and Fourier transforming do not commute. As a consequence, as is readily verified:

$$\phi(\eta, k) = a^3\psi(\eta, k) \tag{21}$$

and in the de Sitter case:

$$\phi(\eta, k) = -\frac{H^3}{\eta^3}\psi(\eta, k) \tag{22}$$

As far as present day observations of cosmological scales are concerned, the distinction between comoving and decoupling momenta does not matter and we therefore obtain from Eq.19:

$$\psi(\eta_h/r, rk) = r^{-3/2}\psi(\eta_h, k) \tag{23}$$

We therefore finally obtain for the fields over comoving momenta as conventionally defined the scaling behavior of the horizon crossing amplitude

$$\psi(\text{horizon crossing}, rk) \sim r^{-3/2} \tag{24}$$

which yields indeed the usual scale invariant spectrum:

$$\langle 0|\psi^\dagger(\text{horizon crossing}, rk)\psi(\text{horizon crossing}, rk)|0\rangle \sim r^{-3} \tag{25}$$

Indeed, this was to be expected because neither the background de Sitter space, nor our introduction of a cutoff, nor the choices of initial conditions (all solutions being obtained from another by mere scaling) broke time translation invariance.

On the other hand, in the case of a non-de Sitter background, the spectrum is of course not scale invariant. In the presence of our cutoff we will then obtain additional scale invariance breaking effects on the spectrum, because of the new cutoff dependent terms in the wave equation.

5 Conclusions

We investigated the signature of the cutoff in the perturbation spectrum from two perspectives, in each case not needing to solve the mode equation explicitly. The adiabatic treatment in Sec. (3) is based on the fact that in order to be consistent with inflation, each mode needs to be in the adiabatic vacuum shortly after the mode is created, whereas the scaling analysis of Sec. (4) utilizes that the wave equation scales trivially and that there is also no reason for the (still unknown) initial conditions to break the (almost) time-translation invariance of the background space-time. Both approaches show that the resulting fluctuation power spectrum is indeed scale invariant if the background space-time is de Sitter (a brief sketch of expected modifications in slow-roll inflationary scenarios can be found in Ref. [5]). The adiabatic analysis, in addition, shows that any corrections of the overall amplitude are at most of order σ^2, where σ is the ratio of the horizon scale and the minimum spatial resolution Δx_{min} admitted by the commutation relation Eq. (1). If Δx_{min} is identified with the Planck length, the CMBR temperature fluctuation amplitude suggests that $\sigma \sim 10^{-5}$. Hence the corrections would be negligibly small if the cutoff scale is indeed the Planck scale.

However, the Planck scale is only the scale at which a natural ultraviolet cutoff may set in the latest. The natural short distance cutoff scale may well be larger, as might be the case e.g. in string theory. The signature of the cutoff in the CMBR would increase if the cutoff scale were larger than the Planck length, e.g. close to the GUT scale. On the other hand, both the scale dependence and the amplitude of the power spectrum are very sensitively dependent on the details of the inflaton potential. Therefore, we cannot make very general conclusions, but can derive an upper bound on Δx_{min} from the observations only after specifying a concrete model for inflation.

An interesting technical question remains: We have not shown how or even if the decoupling modes evolve into the adiabatic vacuum from some natural initial conditions at the singularity. Two possibilities can be imagined: either there exists a symmetry or regularity condition that uniquely specifies initial conditions that later evolve into the adiabatic solution. In this case the discussion in Sec. (3) applies.

Or, alternatively, the modes are generally created in a highly excited state as seen from the point of view of a comoving particle detector. This case would be inconsistent [12, 5] with the assumption of slow-roll inflation made at the onset of Ref. [6]. This would indicate that the combination of short-distance uncertainty of the kind described by Eq. (1) and inflation is not, in general, self-consistent.

We will conclude with some speculative ideas about the first possibility for the initial conditions at the singularity. Starting with the original equation of motion, Eq. (2), expanding the coefficients around $\eta = \eta_c$, and shifting the origin of the time coordinate to η_c, one obtains a differential equation of the form

$$\phi''_{\tilde{k}} - (2\eta)^{-1}\phi'_{\tilde{k}} + \frac{A}{\eta}\phi_{\tilde{k}} = 0 \tag{26}$$

which can be solved analytically:

$$\phi_{\tilde{k}}(\eta) = C_1 F(\eta) + C_2 F(\eta)^* \ , \tag{27}$$

where

$$F(\eta) = \left(\frac{\sqrt{A}}{2} + iA\sqrt{\eta}\right)\exp(-2i\sqrt{A\eta}) \ . \tag{28}$$

The two constants can be specified in formal analogy with the standard procedure by picking the positive "frequency" branch and normalizing according to the Wronskian condition. The result is regular at $\eta = 0$. A preliminary analysis appears to indicate that there exists a unique solution for which $\phi^\dagger\phi$ is analytic at creation time and that it corresponds to this solution. If this solution indeed evolves into the later adiabatic vacuum solution then this would be a desirable intrinsic mechanism for fixing the vacuum.

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